Chapter 6 § 4

Parallel Lines and Proportional Parts

Theorems :

Triangle Proportionality (7-4) – If a line is parallel to one side of a triangle and intersects the other two sides in two distinct points, then it separates these sides into segments of proportional lengths.

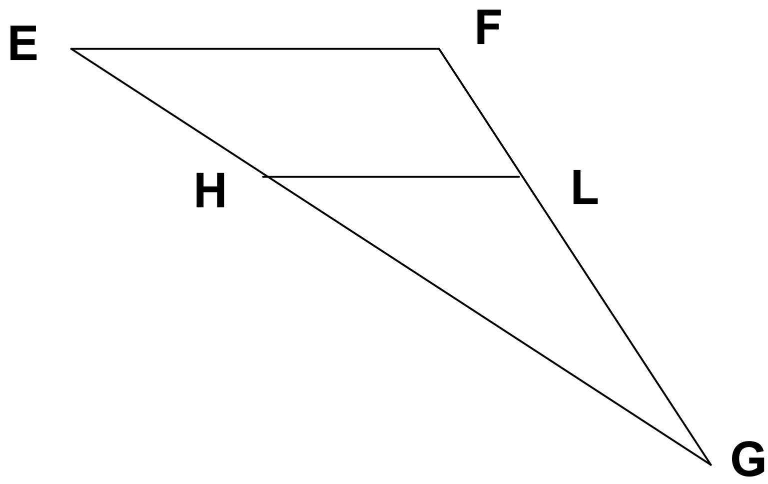
(7-5) – If a line intersects two sides of a triangle and separates the sides into corresponding segments of proportional lengths, then the line is parallel to the third side.

(7-6) – A segment whose endpoints are the midpoints of two sides of a triangle is parallel to the third side of the triangle, and its length is one-half the length of the third side.

Corollaries :

(7-1) – If three or more parallel lines intersect two transversals, then they cut off the transversals proportionally.

(7- 2) – If three or more parallel lines cut off congruent segments on one transversal, then they cut off congruent segments on every transversal.



In ΔEFG, EG = 15, EH = 5, and LG is twice FL. Determine whether HL || EF.

From the segment addition postulate, EG = EH + HG.

EG = EH + HG

15 = 5 + HG

10 = HG

We must show that $\frac{EH}{HG} = \frac{FL}{LG}$

$$\frac{5}{10} = \frac{x}{2x}$$

$$\frac{1}{2} = \frac{1}{2}$$

Since the sides are proportional, then HL || EF.

Triangle ABC has vertices A(0, 2), B(12, 0), and C(2, 10)

a. **Find the coordinates of D, the midpoint of AB, and E, the midpoint of CB.**

b. **Show that DE is parallel to AC.**

c. **Show that 2DE = AC.**

a.

$$(x, y) = \left[\frac{X_1 + X_2}{2}, \frac{Y_1 + Y_2}{2}\right]$$

$$D(AB) = \left[\frac{0 + 12}{2}, \frac{2 + 0}{2}\right] = (6, 1)$$

$$E(CB) = \left[\frac{12 + 2}{2}, \frac{10 + 0}{2}\right] = (7, 5)$$

b. **To show that DE || AC, find the slopes**

$$m = \frac{Y_2 - Y_1}{X_2 - X_1}$$

$$DE = \frac{1 - 5}{6 - 7} = 4 \qquad AC = \frac{2 - 10}{0 - 2} = 4$$

c. To show that 2DE = AC, we use the distance formula.

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$AC = \sqrt{(0 - 2)^2 + (2 - 10)^2} = \sqrt{68}$$

$$DE = \sqrt{(6 - 7)^2 + (1 - 5)^2} = \sqrt{17}$$

Since $2(\sqrt{17}) = \sqrt{68}$ then 2DE = AC