

Chapter 5 § 3

Indirect Proof and inequalities

Definitions :

Indirect reasoning – the type of reasoning that assumes the conclusion is false and then shows that this assumption leads to a contradiction of the hypothesis or some other accepted fact, like a postulate, theorem or corollary.

Indirect proof – the process of indirect reasoning.

Steps for Writing an Indirect Proof

1. Assume that the conclusion is false.
2. Show that the assumption leads to a contradiction of the hypothesis or some other fact such as a postulate, theorem, or corollary.
3. Point out the assumption must be false and, therefore the conclusion must be true.

Examples

State the assumption you would make to start an indirect proof of each statement.

a. The number 117 is divisible by 13.

The number 117 is not divisible by 13.

b. Harrison Ford is the best candidate for the election.

Harrison Ford is not the best candidate for the election.

c. \overline{AB} is the median of $\triangle ABC$.

\overline{AB} is not the median of $\triangle ABC$

d. $m\angle C < m\angle D$

$m\angle C \geq m\angle D$

Given: $\frac{1}{2y + 4} = 20$

Prove: $y \neq -2$

Using Indirect Proof

Step 1: Assume that $y = 2$

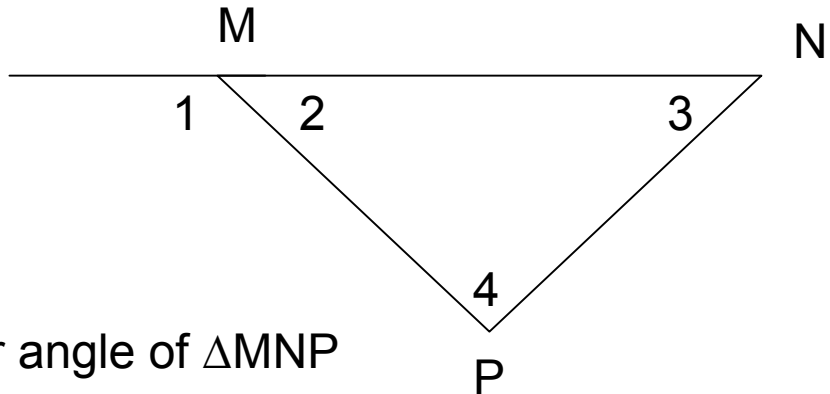
Step 2:

By substituting -2 in for y in the equation and find that
 $2(-2) + 4 = 0$. The denominator cannot be 0 .
So $y \neq -2$.

Step 3:

The assumption leads to a contradiction of a known fact. Since the denominator cannot be 0 the assumption must be false.

Proof of Theorem 5-8



Given: $\angle 1$ is an exterior angle of $\triangle MNP$

Prove: $m\angle 1 > m\angle 4$
 $m\angle 1 > m\angle 3$

Step 1: $m\angle 1 \leq m\angle 4$ and $m\angle 1 \leq m\angle 3$

Step 2:

Case 1: If $m\angle 1 = m\angle 3$

Since $m\angle 1 = m\angle 3 + m\angle 4$ by the Exterior Angle Theorem, then $m\angle 3 = m\angle 3 + m\angle 4$ by substitution. Then $m\angle 4 = 0$, which contradicts the fact that the measure of an angle is greater than zero.

Case 2: If $m\angle 1 < m\angle 3$

By the Exterior Angle Theorem, $m\angle 1 = m\angle 3 + m\angle 4$. Since the angle measures are positive, the definition of inequality implies $m\angle 1 > m\angle 3$ and $m\angle 1 > m\angle 4$. This contradicts the assumption.

Step 3:

In both cases the assumption leads to a contradiction of a known fact. Therefore the assumption must be false.