Chapter 6 § 5

Solving Open Sentences Involving Absolute Value

Definition

Absolute Value – The distance between the specified number and zero.

Note :

If we have a statement $|x| = 4$, then the value of x could be either 4 or -4.

Example

$|x - 3| = 5$

$|x + (-3)| = 5$

Since this is an absolute value we write the two possible forms.

$x + (-3) = 5$	$x + (-3) = -5$
$\quad\quad 3 \quad\quad 3$	$\quad\quad 3 \quad\quad 3$
$x + 0 = 8$	$x + 0 = -2$
$x = 8$	$x = -2$

The solution set is { 8 , -2 }

$$| 4p - 3 | = 7$$

$$| 4p + (-3) | = 7$$

$$4p + (-3) = 7$$

$$\quad\quad 3 \quad\quad 3$$

$$\frac{4p}{4} + 0 = \frac{10}{4}$$

$$p = \frac{5}{2}$$

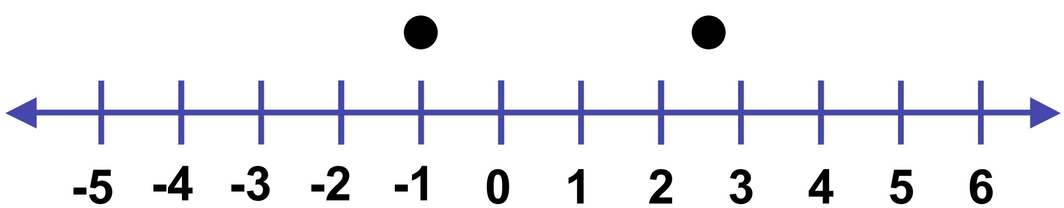
$$4p + (-3) = -7$$

$$\quad\quad 3 \quad\quad 3$$

$$\frac{4p + 0}{4} = \frac{-4}{4}$$

$$p = -1$$

The solution set is $\{ \frac{5}{2}, -1 \}$



$$|2k + 8| = 2$$

$$2k + 8 = 2$$

$$-8 \quad -8$$

$$2k + 0 = -6$$

$$\frac{2k}{2} = \frac{-6}{2}$$

$$k = -3$$

$$2k + 8 = -2$$

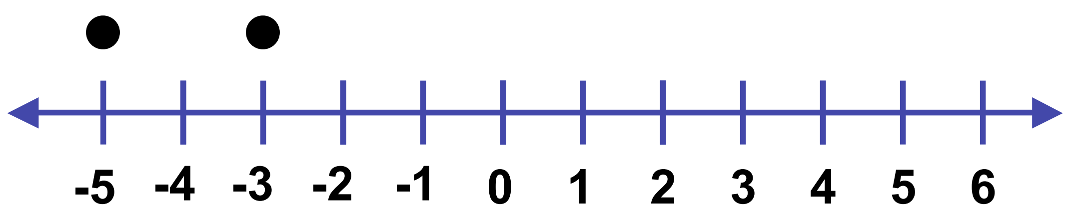
$$-8 \quad -8$$

$$2k + 0 = -10$$

$$\frac{2k}{2} = \frac{-10}{2}$$

$$k = -5$$

The solution set is { -3 , -5 }



Let's Season with Garlic

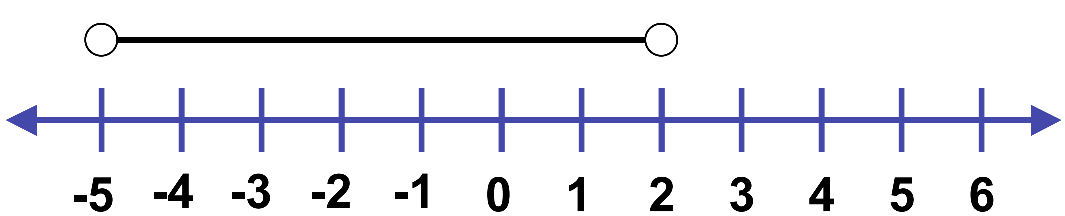
$$|3 + 2x| < 7$$

Note the difference with the inequality and the sign of the right side.

$$3 + 2x < 7$$
$$-3 \qquad\qquad -3$$
$$0 + 2x < 4$$
$$\frac{2x}{2} < \frac{4}{2}$$
$$x < 2$$

$$3 + 2x > -7$$
$$-3 \qquad\qquad -3$$
$$0 + 2x > -10$$
$$\frac{2x}{2} > \frac{-10}{2}$$
$$x > -5$$

The solution set is $\{ -5 < x < 2 \}$



$$|2x + 1| \leq 8$$

$$\begin{aligned} 2x + 1 &\leq 8 \\ -1 \quad & \quad -1 \\ \hline 2x + 0 &\leq 7 \end{aligned}$$

$$\frac{2x}{2} + 0 \leq \frac{7}{2}$$

$$x \leq \frac{7}{2}$$

$$\begin{aligned} 2x + 1 &\geq -8 \\ -1 \quad & \quad -1 \\ \hline 2x + 0 &\geq -9 \end{aligned}$$

$$\frac{2x}{2} + 0 \geq \frac{-9}{2}$$

$$x \geq \frac{-9}{2}$$

The solution set is $\left\{ \frac{-9}{2} \leq x \leq \frac{7}{2} \right\}$