Chapter 5 § 3

Simulating Experiments

Definitions:

Probability model – a model used to calculate a theoretical answer.

Simulation – The imitation of chance behavior, based on a model that accurately reflects the experiment under consideration.

Note:

The steps of a simulation are:

1. State the problem or describe the experiment.
2. State the assumptions
3. Assign digits to represent outcomes.
4. Simulate many repetitions
5. State your conclusions.

Simulation is an effective tool for finding likelihoods of complex results once we have a trustworthy model. In particular, we can use random digits from a table, graphing calculator, or computer software to simulate many repetitions quickly. The proportion of repetitions on which a result occurs will eventually be close to its true likelihood, of simulation can give good estimates of probabilities. The art of random-digit simulation can be illustrated by a series of examples.

Let's refer to the example on page 288.

Step 1 – State the problem or describe the experiment.

Toss a coin 10 times. What is the likelihood of a run of at least 3 consecutive heads or 3 consecutive tails?

Step 2 – State the assumption.

- A head or tail s equally likely to occur on each toss.
- Tosses are independent of each other (i.e. what happens on one toss will not influence the next toss).

Step 3 – Assign digits to represent outcomes.

In a random number table, such as Table B in the back of the book, the digits 0,1,2,3,4,5,6,7,8, and 9 occur with the same long-term relative frequency (1/10). We also know that the successive digits in the table are independent. It follows that even digits and odd digits occur with the same long-term relative frequency, 50%. Here is one assignment of digits for coin tossing:

-One digit simulates one toss of the coin

-Odd digits represent heads; even digits represent tails.

Step 4 – Simulate many repetitions

Looking at 10 consecutive digits in Table B simulate one repetition. Read many groups of 10 digits from the table to simulate many repetitions. Be sure to keep track of whether or not the event we want (a run of 3 heads or 3 tails) occurs on each repetition.

Here is the first 4 simulations starting at line 111 in Table B

81486	**69487**	**60513**	**09297**	**00412**	**71238**	**27649**	**36950**
TH<u>TT</u>	**THTTH**	**TT<u>HH</u>**	**THTHH**	**<u>TTT</u>HT**	**HHTHT**	**THTTH**	**HTHHT**
Yes		Yes		Yes		No	

22 additional repetitions were done for a total of 25 repetitions; 23 of them did have a run of 3 or more heads or tails.

Step 5 – State your conclusions.

We estimate the probability of a run by the proportion:

$$\text{estimated probability} = \frac{23}{25} = 0.92$$

Of course, 25 repetitions are not enough to be confident that our estimate is accurate. Now that we understand how to do the simulations, we can tell a computer to do many thousands of repetitions. A long simulation (or mathematical analysis) finds that the true probability is about 0.826.